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Space-like D_p branes: accelerating cosmologies versus conformally de Sitter space-time

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ABSTRACT: We consider the space-like D_p brane solutions of type II string theories having isometries $ISO(p+1) \times SO(8-p, 1)$. These are asymptotically flat solutions or in other words, the metrics become flat at the time scale $\tau \gg \tau_0$. On the other hand, when $\tau \sim \tau_0$, we get $(p+1)+1$ dimensional flat FLRW metrics upon compactification on a $(8-p)$ dimensional hyperbolic space with time dependent radii. We show that the resultant $(p+1)+1$ dimensional metrics describe transient accelerating cosmologies for all p from 1 to 6, i.e., from $(2+1)$ to $(7+1)$ space-time dimensions. We show how the accelerating phase changes with the interplay of the various parameters characterizing the solutions in $(3+1)$ dimensions. Finally, for $\tau \ll \tau_0$, after compactification on $(8-p)$ dimensional hyperbolic space, the resultant metrics are shown to take the form of $(p+1)+1$ dimensional de Sitter spaces upto a conformal transformation. Cosmologies here are decelerating, but, only in a particular conformal frame we get eternal acceleration.

KEYWORDS: dS vacua in string theory, D-branes

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1 Introduction

S(space)-like branes are topological defects localized on a space-like hypersurface which exist as time dependent solutions of many field theories as well as of string/M theory [1, 2]. In string theory just like Dp branes arise as space-like tachyonic kink solution of world volume field theory of non-BPS $D(p+1)$ brane or $D(p+2)$ — anti $D(p+2)$ brane [3], space-like Dp (or SDp) branes arise as the time-like tachyonic kink solution of the above unstable brane systems [1, 4]. SDp -branes have $(p+1)$ dimensional Euclidean world-volume and carry the same RR charges as their time-like cousins. The original motivation for studying SDp -branes was to understand holography in the temporal context. Just as Dp branes give rise to a space-like direction from a Lorentzian world-volume field theory, SDp branes give rise to a time-like direction from the Euclidean world-volume theory of SDp branes and this is a necessary ingredient for dS/CFT correspondence [5]. One of the reasons for the space-time construction of these SDp branes¹ was to understand the so-called dS/CFT correspondence.

In a previous paper [10] we constructed an anisotropic (in one direction) $SD3$ brane solution of type IIB string theory and compactified on a six dimensional product space of the form $H_5 \times S^1$, where H_5 is a five dimensional hyperbolic space² and S^1 is a circle. The resulting external space was then shown to be conformal to a four dimensional de Sitter space. This brought out the connection between $SD3$ brane and the four dimensional de Sitter space which may be helpful in understanding dS/CFT correspondence [5] in the same spirit as AdS/CFT correspondence [14]. It may be of interest to see if similar structure exists for other SDp branes for $p \neq 3$. Moreover, it is well-known that S-brane solutions of string/M theory give rise to four dimensional accelerating cosmologies (similar to the acceleration of our universe observed in the present epoch [15–17]) upon time dependent hyperbolic space compactification [18–22] and we have seen this, in particular, for $SD2$ -brane compactified on six dimensional hyperbolic space and expressing the resultant metric

¹The space-time constructions of S-branes were given in [6–9].

²Hyperbolic space compactifications are discussed in [11–13].

in Einstein frame [20, 23]. It would be of interest to see whether similar accelerating cosmologies can be obtained in other dimensions and under what conditions.

Motivated by this, we construct in this paper the isotropic SDp brane solutions having isometries $ISO(p+1) \times SO(8-p, 1)$, from the double Wick rotation of the static, non-supersymmetric, charged Dp brane solutions [24] of type II string theories. The isotropic SDp brane solutions will be characterized by three independent parameters $(\tau_0, \theta, \delta_0)$. The parameter τ_0 sets a time scale in the sense that when $\tau \gg \tau_0$, the solutions become flat. On the other hand when $\tau \sim \tau_0$, the isotropic SDp brane metrics can be compactified on $(8-p)$ dimensional hyperbolic spaces of time dependent radii, to obtain a $(p+1)+1$ dimensional flat FLRW metrics in the Einstein frame. We show that these resultant metrics give rise to transient accelerating cosmologies for all p (where $1 \leq p \leq 6$) i.e., from $(2+1)$ to $(7+1)$ space-time dimensions. The amount of acceleration and the duration vary with the variations of the various parameters and we study them only in realistic $(3+1)$ space-time dimensions. When $\tau \ll \tau_0$, we will fix the parameter δ_0 for calculational simplicity (without loss of any generality) and find after a similar compactification on $(8-p)$ dimensional hyperbolic spaces that the resultant metrics can be cast into de Sitter forms in $(p+1)+1$ dimensions upto a conformal factor after a suitable coordinate transformation. This clarifies the relation between SDp branes and de Sitter spaces. The other two parameters θ and δ_0 in the solutions are related to the charge of SDp branes and the dilaton, respectively.

Here we briefly mention that the isotropic SDp brane solutions described in section 2 are not new and they are just rewriting the already known solutions [9, 24, 25] in a convenient form. The accelerating cosmologies were known [19, 20, 23] to follow from the $SD2$ brane solutions upon time dependent hyperbolic space compactification. In this paper we show that similar accelerating cosmologies also follow from all the SDp brane solutions (for $1 \leq p \leq 6$) upon similar time dependent hyperbolic space compactification and is described in section 3. The results of this section are new. Also, a four dimensional de Sitter space upto a conformal factor was obtained before in [10] from the near horizon limit of an anisotropic $SD3$ brane solution of type IIB string theory upon hyperbolic space compactification. In this paper we show that de Sitter solutions upto a conformal factor in $(p+1)+1$ dimensions also follow from all the isotropic SDp brane solutions of type II string theory upon hyperbolic space compactification. This is described in section 4 and here also the results are new. So, the accelerating cosmologies and also the conformally de Sitter solutions are not specific to a particular SDp brane (that were known before), but they are quite generic, as we show in this paper, for all the SDp branes for $1 \leq p \leq 6$.

This paper is organized as follows. In the next section, we give the construction of isotropic SDp brane solutions of type II string theories and write them in a suitable coordinate. In section 3, we show how FLRW type cosmological solutions in various dimensions can be obtained from the isotropic SDp brane solutions by compactifications. We also discuss about the solutions in various dimensions. In section 4, we show how the same solutions give rise to $(p+1)+1$ dimensional de Sitter spaces upto conformal factors in early times. Finally, we conclude in section 5.

2 Isotropic SD p brane solutions

In this section we will give the construction of isotropic SD p brane solutions of type II string theories characterized by three independent parameters and write them in a suitable coordinate system for the ease of our discussion in the next two sections. These solutions can actually be obtained either from the static, non-supersymmetric, isotropic p -brane solutions in arbitrary space-time dimensions given in [24] and using a double Wick rotation, or from the isotropic S-brane solutions in arbitrary dimensions given in [9]. But for convenience we will use the solutions given in eq. (4) of ref. [25], representing nonsupersymmetric intersecting brane solutions involving charged D p branes, and chargeless D1 branes and D0 branes. These solutions contain several parameters and to obtain isotropic nonsupersymmetric D p brane solutions from here we will put the conditions $\delta_2 = \delta_0$, $\bar{\delta} = (p/4)\delta_0$ and also $\delta_1 = -2\delta_0$. The solutions eq. (4) of [25], then take the form,

$$\begin{aligned} ds^2 &= F(r)^{\frac{p+1}{8}} \left(H(r) \tilde{H}(r) \right)^{\frac{2}{7-p}} \left(\frac{H(r)}{\tilde{H}(r)} \right)^{-\frac{p(p+1)}{4(7-p)}\delta_0} (dr^2 + r^2 d\Omega_{8-p}^2) \\ &\quad + F(r)^{-\frac{7-p}{8}} \left(\frac{H(r)}{\tilde{H}(r)} \right)^{\frac{p}{4}\delta_0} \left(-dt^2 + \sum_{i=1}^p (dx^i)^2 \right) \\ e^{2(\phi-\phi_0)} &= F(r)^{\frac{3-p}{2}} \left(\frac{H(r)}{\tilde{H}(r)} \right)^{-(4+p)\delta_0}, \quad F_{[8-p]} = Q \text{Vol}(\Omega_{8-p}) \end{aligned} \quad (2.1)$$

We remark that the other two references mentioned above also give the same solutions, but the parameter relations are simpler here. Note that the metrics in (2.1) are given in the Einstein frame. The various functions appearing in the solutions are defined as,

$$\begin{aligned} F(r) &= \left(\frac{H(r)}{\tilde{H}(r)} \right)^\alpha \cosh^2 \theta - \left(\frac{\tilde{H}(r)}{H(r)} \right)^\beta \sinh^2 \theta \\ H(r) &= 1 + \frac{\omega^{7-p}}{r^{7-p}}, \quad \tilde{H}(r) = 1 - \frac{\omega^{7-p}}{r^{7-p}} \end{aligned} \quad (2.2)$$

There are six parameters α , β , δ_0 , θ , ω , and Q associated with the solutions. However, from the equations of motion, the parameters can be seen to satisfy the following three relations,

$$\begin{aligned} \alpha - \beta &= 3\delta_0 \\ \frac{14+5p}{7-p}\delta_0^2 + \frac{1}{2}\alpha(\alpha - 3\delta_0) &= \frac{8-p}{7-p} \\ Q &= (7-p)\omega^{7-p}(\alpha + \beta) \sinh 2\theta \end{aligned} \quad (2.3)$$

Using these relations we can eliminate three parameters out of the six we mentioned above and therefore, the solutions have three independent parameters, namely, ω , θ and δ_0 . Note from the form of $\tilde{H}(r)$ in (2.2) that the solutions have curvature singularities at $r = \omega$ and therefore, the solutions are well defined only for $r > \omega$. Also in (2.1) ϕ_0 denotes the asymptotic value of the dilaton and $F_{[8-p]}$ is the $(8-p)$ form and Q is the charge

associated with the Dp branes which in this case are magnetically charged. We point out that the singularities at $r = \omega$ are naked singularities where the dilaton becomes plus or minus infinity (depending on the parameters and the value of p) and can not be removed by coordinate transformations or going to a different (dual) frame. However, since these are string theory solutions it may be plausible that these singularities will go away when the higher order stringy effects are taken into account. As far as we know the status of these singularities in full string theory is still not clearly understood.

Now in order to get isotropic SDp brane solutions we apply the double Wick rotation [24] $r \rightarrow it$, $t \rightarrow -ix^{p+1}$ to the solutions (2.1) along with $\omega \rightarrow i\omega$, $\theta \rightarrow i\theta$ and $\theta_1 \rightarrow i\theta_1$, where θ_1 is one of the angles parameterizing the sphere Ω_{8-p} and then we obtain,

$$\begin{aligned}
 ds^2 &= F(t)^{\frac{p+1}{8}} \left(H(t) \tilde{H}(t) \right)^{\frac{2}{7-p}} \left(\frac{H(r)}{\tilde{H}(r)} \right)^{-\frac{p(p+1)}{4(7-p)}\delta_0} (-dt^2 + t^2 dH_{8-p}^2) \\
 &\quad + F(t)^{-\frac{7-p}{8}} \left(\frac{H(t)}{\tilde{H}(t)} \right)^{\frac{p}{4}\delta_0} \sum_{i=1}^{p+1} (dx^i)^2 \\
 e^{2(\phi-\phi_0)} &= F(t)^{\frac{3-p}{2}} \left(\frac{H(t)}{\tilde{H}(t)} \right)^{-(4+p)\delta_0}, \quad F_{[8-p]} = (-1)^{8-p} Q \text{Vol}(H_{8-p}) \quad (2.4)
 \end{aligned}$$

where the various functions are now given as,

$$\begin{aligned}
 F(t) &= \left(\frac{H(t)}{\tilde{H}(t)} \right)^\alpha \cos^2 \theta + \left(\frac{\tilde{H}(t)}{H(t)} \right)^\beta \sin^2 \theta \\
 H(t) &= 1 + \frac{\omega^{7-p}}{t^{7-p}}, \quad \tilde{H}(t) = 1 - \frac{\omega^{7-p}}{t^{7-p}} \quad (2.5)
 \end{aligned}$$

Note that under the Wick rotation the solutions have become time dependent. The naked singularities at $r = \omega$ has now changed to the singularities at $t = \omega$. Also, the metric of the sphere $d\Omega_{8-p}^2$ has changed to negative of the metric of the hyperbolic space dH_{8-p}^2 . The metrics now has the symmetry $\text{ISO}(p+1) \times \text{SO}(8-p, 1)$. The hyperbolic functions $\sinh \theta$, $\cosh \theta$ have become trigonometric functions and the function F has relative plus sign in the two terms instead of minus. Most importantly the form field remains real and retains its form upto a sign which does not happen for the BPS Dp branes (Wick rotation actually makes the form field imaginary for BPS Dp branes and the solutions in that case do not remain solutions of type II theories, instead they become solutions of type II* theories [26]). The first two parameter relations in (2.3) remain the same under the Wick rotation, whereas the last relation changes to $Q = (7-p)\omega^{7-p}(\alpha+\beta)\sin 2\theta$ if we insist that Q should also change under the Wick rotation as $Q \rightarrow (i)^{8-p}Q$. Eq. (2.4) represents real isotropic SDp brane solutions of type II string theories characterized by three independent parameters ω , θ , δ_0 .

Now for the discussion in the next two sections we will make a coordinate transformation from t to τ given by,

$$t = \tau \left(\frac{1 + \sqrt{g(\tau)}}{2} \right)^{\frac{2}{7-p}}, \quad \text{where,} \quad g(\tau) = 1 + \frac{4\omega^{7-p}}{\tau^{7-p}} \equiv 1 + \frac{\tau_0^{7-p}}{\tau^{7-p}} \quad (2.6)$$

Under this coordinate change we get,

$$\begin{aligned}
 H(t) &= 1 + \frac{\omega^{7-p}}{t^{7-p}} = \frac{2\sqrt{g(\tau)}}{1 + \sqrt{g(\tau)}}, & \tilde{H}(t) &= 1 - \frac{\omega^{7-p}}{t^{7-p}} = \frac{2}{1 + \sqrt{g(\tau)}}, \\
 H(t)\tilde{H}(t) &= \frac{4\sqrt{g(\tau)}}{(1 + \sqrt{g(\tau)})^2}, & \frac{H(t)}{\tilde{H}(t)} &= \sqrt{g(\tau)}, \\
 -dt^2 + t^2 dH_{8-p}^2 &= g(\tau)^{\frac{1}{7-p}} \left(-\frac{d\tau^2}{g(\tau)} + \tau^2 dH_{8-p}^2 \right)
 \end{aligned} \tag{2.7}$$

Using these relations we can rewrite the isotropic SD p brane solutions given in (2.4) as follows,

$$\begin{aligned}
 ds^2 &= F(\tau)^{\frac{p+1}{8}} g(\tau)^{\frac{1}{7-p} - \frac{p(p+1)}{8(7-p)} \delta_0} \left(-\frac{d\tau^2}{g(\tau)} + \tau^2 dH_{8-p}^2 \right) + F(\tau)^{-\frac{7-p}{8}} g(\tau)^{\frac{p}{8} \delta_0} \sum_{i=1}^{p+1} (dx^i)^2 \\
 e^{2(\phi-\phi_0)} &= F(\tau)^{\frac{3-p}{2}} g(\tau)^{-\frac{(4+p)}{2} \delta_0}, & F_{[8-p]} &= (-1)^{8-p} Q \text{Vol}(H_{8-p})
 \end{aligned} \tag{2.8}$$

where $g(\tau)$ is as given in (2.6) and $F(\tau)$ is given by,

$$F(\tau) = g(\tau)^{\frac{\alpha}{2}} \cos^2 \theta + g(\tau)^{-\frac{\beta}{2}} \sin^2 \theta \tag{2.9}$$

The parameter relations remain the same as given in (2.3) with the factor $\sinh 2\theta$ in the last one replaced by $\sin 2\theta$. It should be noted from (2.8), that in the new coordinate, the original singularities at $t = \omega$ have been shifted to $\tau = 0$. Now the solutions have three independent parameters, namely, τ_0 , θ , δ_0 . Also note that as $\tau \gg \tau_0$, $g(\tau)$, $F(\tau) \rightarrow 1$ and therefore, the solutions reduce to flat space. In the next two sections we will use the solutions (2.8) to see how one can get cosmologies in various dimensions and also how to obtain de Sitter spaces upto a conformal factor.

3 FLRW cosmologies from SD p brane compactifications

In this section we will see how we can get flat FLRW cosmologies in various dimensions from the isotropic SD p brane solutions given in (2.8). We will assume that $\tau \sim \tau_0$, so that the two terms in the function $g(\tau) = 1 + \tau_0^{7-p}/\tau^{7-p}$ are comparable and we must keep both the terms. Keeping this in mind we can rewrite the metrics in (2.8) in the following form,

$$ds^2 = F(\tau)^{-\frac{(p+1)(8-p)}{8p}} g(\tau)^{-\frac{8-p}{p(7-p)} + \frac{(8-p)(p+1)}{8(7-p)} \delta_0} \tau^{-\frac{2(8-p)}{p}} ds_E^2 + F(\tau)^{\frac{p+1}{8}} g(\tau)^{\frac{1}{7-p} - \frac{p(p+1)}{8(7-p)} \delta_0} \tau^2 dH_{8-p}^2 \tag{3.1}$$

where

$$ds_E^2 = -F(\tau)^{\frac{p+1}{p}} g(\tau)^{\frac{8}{p(7-p)} - \frac{p+1}{7-p} \delta_0 - 1} \tau^{\frac{2(8-p)}{p}} d\tau^2 + F(\tau)^{\frac{1}{p}} g(\tau)^{\frac{8-p}{p(7-p)} - \frac{1}{7-p} \delta_0} \tau^{\frac{2(8-p)}{p}} \sum_{i=1}^{p+1} (dx^i)^2 \tag{3.2}$$

is the $(p+1) + 1$ dimensional metrics in the Einstein frame. One can think of these metrics as coming from the compactification of the ten dimensional metrics (3.1) on $(8-p)$ dimensional hyperbolic space with time dependent radius given by

$$R(\tau) = F(\tau)^{\frac{p+1}{16}} g(\tau)^{\frac{1}{2(7-p)} - \frac{p(p+1)}{16(7-p)} \delta_0} \tau \tag{3.3}$$

and expressing the resulting metrics in the Einstein frame. Now defining a new time coordinate η by,

$$d\eta = F(\tau)^{\frac{p+1}{2p}} g(\tau)^{\frac{4}{p(7-p)} - \frac{p+1}{2(7-p)}} \delta_0^{-\frac{1}{2}} \tau^{\frac{8-p}{p}} d\tau \equiv C(\tau) d\tau \quad (3.4)$$

we can rewrite the Einstein frame metrics ds_E^2 in the standard flat FLRW form in $(p+1)+1$ dimensions as

$$ds_E^2 = -d\eta^2 + S^2(\eta) \sum_{i=1}^{p+1} (dx^i)^2 \quad (3.5)$$

where the scale factor $S(\eta)$ is given by,

$$S(\eta) \equiv A(\tau) = F(\tau)^{\frac{1}{2p}} g(\tau)^{\frac{8-p}{2p(7-p)} - \frac{1}{2(7-p)}} \delta_0^{\frac{8-p}{p}} \tau^{\frac{8-p}{p}} \quad (3.6)$$

Now we define another function

$$B(\tau) = \frac{A(\tau)}{C(\tau)} = F(\tau)^{-\frac{1}{2}} g(\tau)^{\frac{6-p}{2(7-p)} + \frac{p}{2(7-p)}} \delta_0 \quad (3.7)$$

where $C(\tau)$ is defined in (3.4). Now the universe is expanding if the scale factor $S(\eta)$ satisfies $dS(\eta)/d\eta > 0$ and the expansion is accelerating if it further satisfies $d^2S(\eta)/d\eta^2 > 0$. Since $S(\eta)$ is a complicated function of η , we will translate [23] these two conditions in terms of the two known functions $A(\tau)$ and $B(\tau)$ given in (3.6) and (3.7). The conditions are,

$$\begin{aligned} m(\tau) &\equiv \frac{d \ln A(\tau)}{d\tau} > 0 \\ n(\tau) &\equiv \frac{d^2 \ln A(\tau)}{d\tau^2} + \frac{d \ln A(\tau)}{d\tau} \frac{d \ln B(\tau)}{d\tau} > 0 \end{aligned} \quad (3.8)$$

where $m(\tau)$ is the expansion parameter and $n(\tau)$ is the rate of expansion parameter. The parameters α and β which appear in the definition of $F(\tau)$ given in (2.9) can be given in terms of δ_0 from the second relation in (2.3) as,

$$\begin{aligned} \alpha &= \frac{3}{2} \delta_0 \pm \sqrt{\frac{8(8-p) - 49(p+1)\delta_0^2}{4(7-p)}} \\ \beta &= -\frac{3}{2} \delta_0 \pm \sqrt{\frac{8(8-p) - 49(p+1)\delta_0^2}{4(7-p)}} \end{aligned} \quad (3.9)$$

In figures 1, 2, we have plotted the expansion parameter $m(\tau)$ and the rate of expansion parameter $n(\tau)$, respectively. We have used the $(p+1)+1$ dimensional metrics ds_E^2 given in (3.2) and the functions $A(\tau)$ and $B(\tau)$ given in (3.6) and (3.7). We have also used the value of α, β given in (3.9). The positive value of $m(\tau)$ in figure 1 indicates the expansion of the universe. From the above plot, we see that the universe expands for all values of p (where $1 \leq p \leq 6$) and therefore, we get the expanding $2+1, 3+1$ upto $7+1$ dimensional universes. In figure 1, the values of the various parameters we have chosen are $\theta = 0$ (this means that the form field is zero and therefore the solution is chargeless and simpler), $\tau_0 = 1$ (this is a typical value we have chosen to show the cosmologies in various dimensions

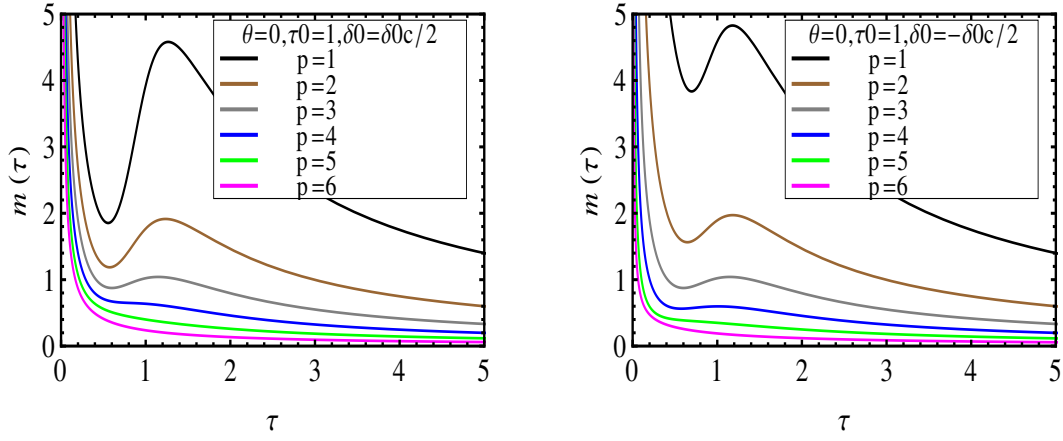


Figure 1. Plot of expansion parameter $m(\tau)$ for SDp brane compactifications on hyperbolic space H_{8-p} given in (3.8). The functions are plotted for $\theta = 0$, $\tau_0 = 1$, $\delta_0 = \frac{\delta_{0c}}{2} = \frac{1}{7}\sqrt{\frac{2(8-p)}{p+1}}$ and $p = 1, \dots, 6$ in the left panel and for $\theta = 0$, $\tau_0 = 1$, $\delta_0 = -\frac{\delta_{0c}}{2} = -\frac{1}{7}\sqrt{\frac{2(8-p)}{p+1}}$ and $p = 1, \dots, 6$ in the right panel.

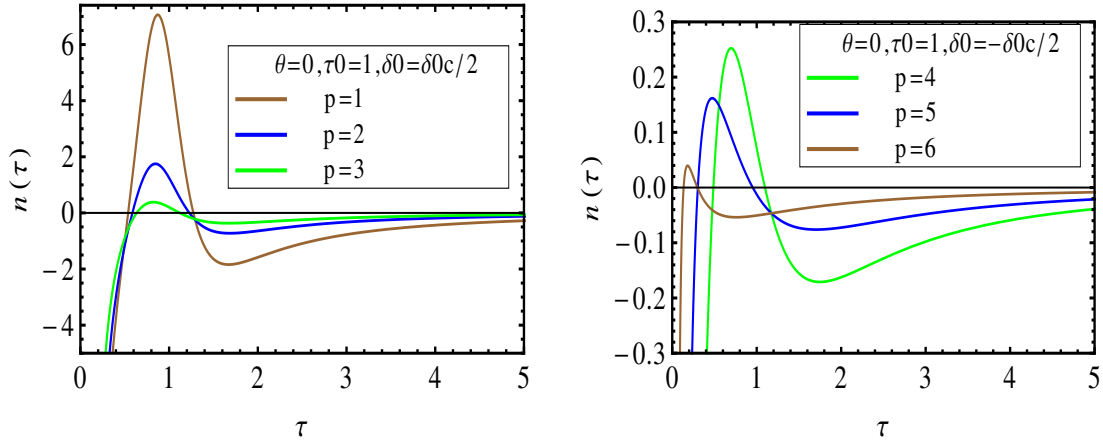


Figure 2. Plot of rate of expansion parameter $n(\tau)$ for SDp brane compactifications on hyperbolic space H_{8-p} given in (3.8). Here the functions are plotted for $\theta = 0$, $\tau_0 = 1$, $\delta_0 = \frac{\delta_{0c}}{2} = \frac{1}{7}\sqrt{\frac{2(8-p)}{p+1}}$ and $p = 1, 2, 3$ in the left panel and $\theta = 0$, $\tau_0 = 1$, $\delta_0 = -\frac{\delta_{0c}}{2} = -\frac{1}{7}\sqrt{\frac{2(8-p)}{p+1}}$ and $p = 4, 5, 6$ in the right panel.

and if τ_0 is less than this value the acceleration is more but the duration is less as seen in figure 5) and $\delta_0 = \delta_{0c}/2$ (defined below) in the left panel and $\delta_0 = -\delta_{0c}/2$ in the right panel. Actually, the parameter δ_0 can not take any arbitrary value. From (3.9) we note that since the parameters α and β are real δ_0 must lie in between

$$-\frac{2}{7}\sqrt{\frac{2(8-p)}{p+1}} \leq \delta_0 \leq \frac{2}{7}\sqrt{\frac{2(8-p)}{p+1}} = \delta_{0c}, \quad (3.10)$$

where we have called the maximum value of δ_0 as δ_{0c} . In figure 1, we have chosen the value

$(p+1)+1$	δ_0	τ	$n(\delta_0, \tau)$
2+1	$\frac{\delta_{0c}}{2}$	0.87197	7.05882
3+1	$\frac{\delta_{0c}}{4}$	0.84025	1.93012
4+1	0	0.78463	0.72178
5+1	$-\frac{\delta_{0c}}{4}$	0.68025	0.30926
6+1	$-\frac{\delta_{0c}}{2}$	0.47336	0.16178
7+1	$-\frac{3\delta_{0c}}{4}$	0.11068	0.32242

Table 1. Here $n(\delta_0, \tau)$ is treated as a function of two variables δ_0 and τ . We have found the particular value of δ_0 which makes the rate of expansion maximum for $\theta = 0$, $\tau_0 = 1$ and $p = 1, \dots, 6$.

of δ_0 as $\pm 1/2$ of its maximum value in the left panel and in the right panel respectively and get expanding universes in all dimensions. The reason for choosing these particular values is that we get accelerating expansion for these values for different p as shown in figure 2. Figure 2 also contains two panels. Here again the positivity of $n(\tau)$ gives an accelerating phase of expansion. On the left panel of figure 2, we show that $n(\tau)$ remains positive for certain interval of time for $p = 1, 2, 3$ and on the right panel we show the positivity of $n(\tau)$ for certain interval of time for $p = 4, 5, 6$. Therefore, we get accelerating expansions for all values of p from 1 to 6. Note that the magnitude of acceleration and the duration depend crucially on the parameters θ , τ_0 and particularly δ_0 . If the parameters are not chosen judiciously, we do not get accelerations. For $p = 1, 2, 3$, we have chosen $\theta = 0$, $\tau_0 = 1$ and $\delta_0 = \delta_{0c}/2$ in the left panel of figure 2 to get accelerations. If we keep the same values of the parameters we get acceleration for $p = 4$ but no accelerations for $p = 5, 6$. This is the reason, for $p = 4, 5, 6$ we have chosen $\theta = 0$, $\tau_0 = 1$ and $\delta_0 = -\delta_{0c}/2$ in the right panel of figure 2 and get accelerations in all the cases. This shows that we can get accelerating cosmologies for all values of p by the appropriate choice of the various parameters characterizing the SD p solutions. The expansion, however, becomes decelerating in the remote past, i.e., for $\tau \ll \tau_0$ and also in the far future $\tau \gg \tau_0$ irrespective of spacetime dimensions and other parameters and all the curves tend to merge in those two regions. We will discuss those cases later. We have tabulated the values of δ_0 for which the rate of expansion parameter $n(\tau)$ is maximum for different values of p in table 1. We have also given those maximum values and the values of τ where these maxima occurs. We have chosen $\theta = 0$ and $\tau_0 = 1$. In all the cases the maximum values are found to be positive and so there are accelerations for all p .

In figures 3, 4, 5 below, we have plotted the rate of expansion parameter $n(\tau)$ for various values of θ , the charge parameter, δ_0 , the dilaton parameter and τ_0 , the time scale, respectively. We have taken $p = 2$, so that the space-time is 3+1 dimensional. In figure 3, we have taken $\tau_0 = 1$ and $\delta_0 = \delta_{0c}/4 = 1/7$. We find that there is acceleration for all values of θ in the range $0 \leq \theta < \pi/2$. The acceleration is minimum for $\theta = 0$ and it gradually increases as we increase the value of θ , except at $\theta = \pi/2$. The duration of the accelerating phase gradually decreases with increasing θ and becomes zero exactly at $\frac{\pi}{2}$. This happens for every dimension where there is an accelerating phase. Note that here we have used the

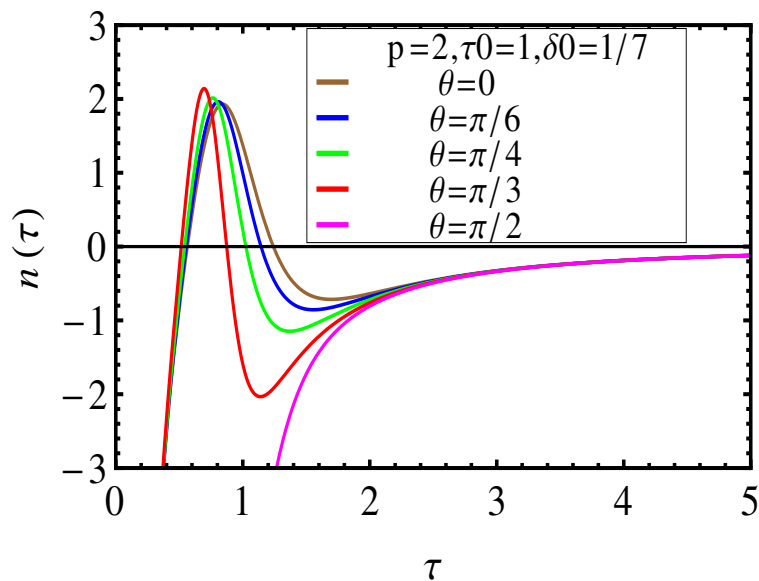


Figure 3. The plot of $n(\tau)$ for different values of θ where $p = 2$, $\tau_0 = 1$ and $\delta_0 = \frac{\delta_{0c}}{4} = \frac{1}{7}$.

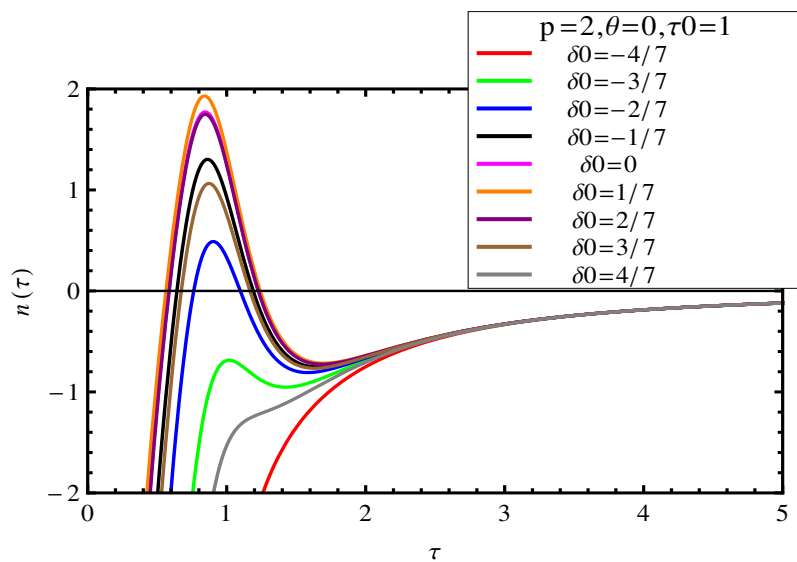


Figure 4. The plot of $n(\tau)$ for $\delta_0 = -\delta_{0c} (= -\frac{4}{7})$, $-\frac{3\delta_{0c}}{4} (= -\frac{3}{7})$, $-\frac{\delta_{0c}}{2} (= -\frac{2}{7})$, $-\frac{\delta_{0c}}{4} (= -\frac{1}{7})$, 0 , $\frac{\delta_{0c}}{4} (= \frac{1}{7})$, $\frac{\delta_{0c}}{2} (= \frac{2}{7})$, $\frac{3\delta_{0c}}{4} (= \frac{3}{7})$, $\delta_{0c} (= \frac{4}{7})$, $p = 2$ and $\tau_0 = 1$.

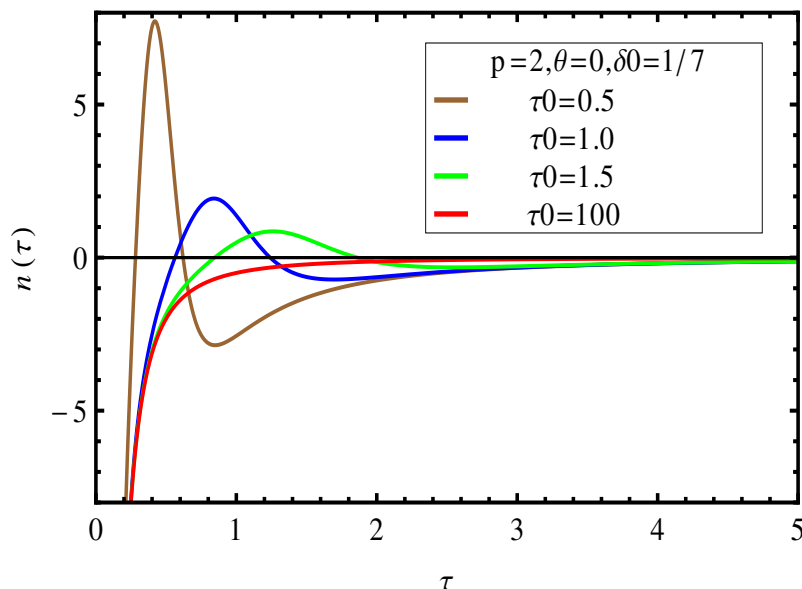


Figure 5. Plot of $n(\tau)$ for different values of τ_0 , with the other parameters kept fixed at $\theta = 0$, $\delta_0 = \frac{\delta_{0c}}{4} = \frac{1}{7}$ and $p = 2$.

upper sign of α given in (3.9). If we use the lower sign we get exactly the same behavior with the interchange of $\theta = 0$ and $\theta = \pi/2$. Similar results can also be obtained for other values of p . In figure 4, we have plotted the rate of expansion parameter $n(\tau)$ for different values of δ_0 , with the other parameters kept fixed at $\theta = 0$ and $\tau_0 = 1$. We have again chosen $p = 2$, corresponding to 3+1 dimensional universe. The accelerating phase depends on the value of δ_0 . We varied δ_0 from δ_{0c} to $-\delta_{0c}$ in an interval of $(1/4)\delta_{0c}$. We find from the figure that the acceleration is maximum for $\delta_0 = \frac{\delta_{0c}}{4}$. Acceleration decreases for other absolute values of δ_0 . For $\delta_0 = 0$ and $\delta_{0c}/2$ we get reasonably large acceleration although plus value gives more acceleration than minus value. On the other hand, for $\delta_0 = \pm\delta_{0c}$ we always get deceleration. Again this happens for each dimension where there is an accelerating phase. As before there exists a critical absolute value of δ_0 for each p above which there is an accelerating phase and below which there will always be deceleration. Similar conclusion can be drawn for other values of p . In figure 5, we have plotted $n(\tau)$ for different values of τ_0 , with the other parameters kept fixed at $\theta = 0$ and $\delta_0 = \delta_{0c}/4$. We have chosen $p = 2$ as before such that we have 3 + 1 dimensional space-time. Here we find that the acceleration is more but of shorter duration as we decrease the value of τ_0 . However, as τ_0 is increased beyond a certain value we always have deceleration. This also happens for each dimension where there is an accelerating phase. There is a critical value of τ_0 at each dimension $p = 1, \dots, 6$, below which there is an accelerating phase and above which there is always a deceleration.

We remark that even though it is difficult to obtain an exact relation between η and τ from the relation (3.4), but it can be integrated in the far future $\tau \gg \tau_0$ and also in the remote past $\tau \ll \tau_0$. The case of $\tau \ll \tau_0$ will be discussed in the next section. Here we mention that for $\tau \gg \tau_0$, τ is related to η by the relation $\tau \sim (\eta - \eta_0)^{\frac{p}{8}}$. Therefore, we have

the scale factor (3.6) to take the form $S(\eta) \sim (\eta - \eta_0)^{1-\frac{p}{8}}$. It is clear from here that in the far future the universe will expand with deceleration for all p as we have seen in figures 1,2.

4 Conformally de Sitter spaces in various dimensions

In this section we will see how at early times $\tau \ll \tau_0$, we can get de Sitter solutions upto a conformal transformation in various dimensions from the SD p solutions given in (2.8). Note that in this case the function $g(\tau)$ can be approximated as,

$$g(\tau) = 1 + \frac{\tau_0^{7-p}}{\tau^{7-p}} \approx \frac{\tau_0^{7-p}}{\tau^{7-p}} \quad (4.1)$$

and so the function $F(\tau)$ given in (2.9) can be approximated as,

$$F(\tau) \approx \left(\frac{\tau_0}{\tau}\right)^{\frac{7-p}{2}\alpha} \cos^2 \theta \quad (4.2)$$

Here we have assumed $\theta \neq \pi/2$, otherwise, it is arbitrary. If $\theta = \pi/2$, then $F(\tau)$ takes the form $F(\tau) = (\frac{\tau_0}{\tau})^{-(7-p)\beta/2}$. But, as we will see that since the final answer will be independent of the parameters α, β , we can take the form of $F(\tau)$ as given in (4.2) without any loss of generality. We will further choose $\alpha + \beta = 2$ for calculational simplicity and again without losing any generality. Now since from the parameter relations given in (2.3) we have $\alpha - \beta = 3\delta_0$, combining these two we get $\alpha = 1 + (3/2)\delta_0$. For more simplification we will set $\tau_0 = 1$ and $\theta = 0$. With all these the metric and the dilaton in (2.8) take the forms,

$$\begin{aligned} ds^2 = & -\tau^{\frac{(7-p)(15-p)-16}{16} - \frac{7(3-p)(p+1)}{32}\delta_0} d\tau^2 + \tau^{\frac{(7-p)^2}{16} + \frac{7(7-p)(3-p)}{32}\delta_0} \sum_{i=1}^{p+1} (dx^i)^2 \\ & + \tau^{\frac{16-(7-p)(p+1)}{16} - \frac{7(3-p)(p+1)}{32}\delta_0} \delta_0 dH_{8-p}^2 \\ e^{2(\phi-\phi_0)} = & \tau^{-\frac{(3-p)(7-p)}{4} + \frac{7(7-p)(p+1)}{8}\delta_0} \end{aligned} \quad (4.3)$$

It should be mentioned that here δ_0 is not a free parameter, unlike in the previous section where we did not use $\alpha + \beta = 2$. In fact, since $\alpha = 1 + (3/2)\delta_0$, we can use the second parameter relation in (2.3) to obtain the value of δ_0 as,

$$\delta_0 = \pm \frac{2}{7} \sqrt{\frac{9-p}{p+1}} \quad (4.4)$$

Now the metrics in (4.3) can also be written as,

$$ds^2 = - \left(\tau^{\frac{16-(7-p)(p+1)}{16} - \frac{7(3-p)(p+1)}{32}\delta_0} \right)^{\frac{p-8}{p}} ds_E^2 + \tau^{\frac{16-(7-p)(p+1)}{16} - \frac{7(3-p)(p+1)}{32}\delta_0} \delta_0 dH_{8-p}^2 \quad (4.5)$$

where ds_E^2 is a $(p+1) + 1$ dimensional metrics in the Einstein frame and have the forms,

$$ds_E^2 = \tau^{\frac{(9-p)(p+1)}{2p} - \frac{7(3-p)(p+1)}{4p}\delta_0} \left[-\frac{d\tau^2}{\tau^2} + \tau^{-\frac{(9-p)}{2} + \frac{7(3-p)}{4}\delta_0} \sum_{i=1}^{p+1} (dx^i)^2 \right] \quad (4.6)$$

Actually the metrics in (4.6) can be seen to arise from a $(8-p)$ dimensional hyperbolic space compactifications with time dependent radius $R(\tau) = \tau^{\frac{16-(7-p)(p+1)}{8} - \frac{7(3-p)(p+1)}{16}} \delta_0$ and then expressing the resulting $(p+1)+1$ dimensional metrics in the Einstein frame. We notice that for $p=3$, $R(\tau)=1$ and for this case the transverse hyperbolic space H_5 gets decoupled from the rest of the space-time, similar to what happens for D3 brane where the transverse S^5 gets decoupled. This simplification for $p=3$ case occurs because of our particular choice of parameters, namely, $\alpha + \beta = 2$. Defining a canonical time by the relation,

$$\eta^2 = \tau^{\frac{(9-p)}{2} - \frac{7(3-p)}{4}} \delta_0 \quad (4.7)$$

we can rewrite the metrics in (4.6) as,

$$ds_E^2 = \eta^{\frac{2(p+1)}{p}} \left[-\frac{d\eta^2}{\eta^2} + \frac{\sum_{i=1}^{p+1} (dx^i)^2}{\eta^2} \right] \quad (4.8)$$

Note that in writing the above metrics we have scaled η and x^i 's as follows,

$$\begin{aligned} \eta &\rightarrow \left(\frac{9-p}{4} - \frac{7(3-p)}{16} \delta_0 \right)^{\frac{p}{p+1}} \eta \\ x^i &\rightarrow \left(\frac{9-p}{4} - \frac{7(3-p)}{16} \delta_0 \right)^{\frac{p}{p+1}} x^i, \quad \text{for } i = 1, \dots, (p+1) \end{aligned} \quad (4.9)$$

The dilaton given in (4.3) can also be written in terms of canonical time using (4.7). We recognize the metrics in (4.8) to be the de Sitter metrics in $(p+1)+1$ dimensions upto the conformal factor $\eta^{2(p+1)/p}$. For $p=2$, i.e., for the four dimensional case the conformal factor becomes η^3 precisely the form we obtained in [10].

We can compare the situation here with the time-like or static BPS Dp -brane cases. For the usual Dp branes when $p=3$, the near horizon limit gives $\text{AdS}_5 \times S^5$ solution. In other words, compactifying on S^5 , we get AdS_5 solution. So, here the boundary theory is conformally invariant. But for other Dp branes (except for $p=5$), compactifying on S^{8-p} , we do not get AdS_{p+2} spaces directly, but we get them upto a conformal factor. So, the boundary theories for these cases do not have conformal symmetry (as the bulk spaces are not AdS spaces — conformal factors make the bulk spaces to be different from AdS spaces), but still the connection with the AdS spaces upto a conformal transformation proves to be useful for calculational purposes. For, space-like Dp branes compactifying on hyperbolic spaces H_{8-p} , we never get (for any p) de Sitter spaces, but we get them (dS_{p+2}) upto a conformal factor exactly like static Dp branes with $p \neq 3, 5$. Here also the boundary theories do not have conformal symmetry since the bulk solution is not a de Sitter solution. However, this connection with de Sitter solution with the space-like Dp branes might prove to be useful for calculational purposes (for example, calculation of correlation function) as in the static Dp brane cases.

To see that the space-times given in (4.8) describe decelerating expansions we rewrite them in flat FLRW forms by defining a new canonical coordinate by $d\tilde{\eta} = \eta^{\frac{1}{p}} d\eta$. The metrics in (4.8) then takes the forms,

$$ds_E^2 = -d\tilde{\eta}^2 + S^2(\tilde{\eta}) \sum_{i=1}^{p+1} (dx^i)^2 \quad (4.10)$$

where the scale factor is given by $S(\tilde{\eta}) \sim (\tilde{\eta} - \tilde{\eta}_0)^{1-\frac{p}{p+1}}$. This clearly shows that the universes expand with deceleration for all p . For $p = 2$, we get $S(\tilde{\eta}) \sim (\tilde{\eta} - \tilde{\eta}_0)^{\frac{1}{3}}$, the result that was obtained in [18].

Thus we have seen how starting from isotropic SD p brane solutions of type II string theories, we get $(p+1)+1$ dimensional de Sitter spaces upto a conformal factor by compactifying on $(8-p)$ dimensional hyperbolic spaces. This brings out the connection between space-like branes and the de Sitter space which might be helpful in understanding dS/CFT correspondence in the same spirit as AdS/CFT correspondence. From the metrics in this case we find that the space-times undergo decelerating expansion for all p , but only in particular conformal frames we get de Sitter spaces, i.e., eternal accelerations.

5 Conclusion

To conclude, in this paper we have studied the various cosmological scenarios that are obtained from the isotropic space-like D p brane solutions of type II string theories by compactifications on $(8-p)$ dimensional hyperbolic spaces and also found the connection between SD p branes and $(p+1)+1$ dimensional de Sitter spaces. The SD p brane solutions are characterized by three independent parameters τ_0 , θ and δ_0 . τ_0 sets a time scale in the theory, θ is related to the RR charge associated with SD p branes and δ_0 is associated with the dilaton in the sense that when $p = 3$, the dilaton is trivial for $\delta_0 = 0$ much like time-like D3 branes. τ_0 gives a time scale because when $\tau \gg \tau_0$, the SD p brane solutions reduce to flat spaces and in that sense these solutions are asymptotically ($\tau \rightarrow \infty$) flat. At large time or in the far future we found that the external space-times undergo decelerating expansions where the scale factors behave like $S(\eta) \sim (\eta - \eta_0)^{1-\frac{p}{8}}$, for all values of p from 1 to 6. On the other hand, when $\tau \sim \tau_0$, the SD p branes upon compactifications by hyperbolic spaces give external space-times which in suitable coordinate can be recast into flat FLRW forms. Here we kept the parameter δ_0 to be arbitrary and found that $(p+1)+1$ dimensional external spaces undergo accelerating expansions for all p . We have studied various cases numerically; because of the complicated nature of the solutions, it is not possible to study them analytically. We have plotted the expansion parameter $m(\tau)$ and the rate of expansion parameter $n(\tau)$ defined in the text, for various values of p to show the cosmologies in various dimensions. We found that for all p lying between 1 to 6, there is a region where $n(\tau)$ becomes positive for certain finite interval of time indicating that universes undergo a transient phase of accelerating expansion. We have also plotted $n(\tau)$ when we vary the three parameters θ , δ_0 and τ_0 while keeping the other parameters fixed in figures 3, 4, and 5 respectively. These show how the acceleration changes as we vary the parameters. Finally, we have shown that at early time, i.e., for $\tau \ll \tau_0$, the $(p+1)+1$ dimensional external spaces can be cast into the form of de Sitter metrics upto a conformal transformation for all values of p . Here we have fixed the parameter δ_0 for calculational simplicity. This brings out the connection between the SD p branes and the de Sitter space which was the original motivation for constructing the space-like branes, and might be useful in understanding dS/CFT correspondence in the same spirit as AdS/CFT

correspondence. We mentioned that the cosmologies here again are decelerating, but they give eternal accelerations only in a special conformal frame.

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